

2.38.1. Indirect Deduction Problems

A. For the following indirect deduction, supply a **justification** for each line (to the right of that line).

1. $(P \vee Q)$
 2. $(\sim P \vee Q)$
 3. $\sim Q$
 4. P
 5. $\sim P$
 6. Q
- Get: Q (ID)

B. The following argument is clearly **invalid**.

- | | |
|--|--|
| 1. Neko is a cat and Jack is a cat.
<hr style="width: 20%; margin: 0 auto;"/> | 1. $(P \wedge Q)$
<hr style="width: 20%; margin: 0 auto;"/> |
| \therefore 2. Neko is a cat but she's not a cat. | \therefore 2. $(P \wedge \sim P)$ |

But the following deduction seems to deduce the conclusion of this argument from its premise. **Explain what mistake has been made in this deduction.**

1. $(P \wedge Q)$
 2. $\sim P$
 3. P
 4. P
 5. $(P \wedge \sim P)$
- Get: P (ID)
- AID
- 1, $\wedge-$
- 2, 3, ID
- 2, 4, $\wedge+$

C. Recall two features of validity noted earlier.¹

- **Self-Entailment:** each sentence validly entails itself. For instance, “P” follows validly from premise “P”. (This trivial point underlies the equally trivial deductive rule *Repetition*.)
- **Monotonicity:** if an argument is valid, adding additional premises maintains that validity. For example, since “ $(P \vee Q) \cdot \sim P \therefore Q$ ” is valid, so is “ $X \cdot (P \vee Q) \cdot \sim P \therefore Q$ ”. (Simply adding premises, however irrelevant, can never turn a valid argument into an invalid one.)

Now a valid argument is (trivially) one whose premises validly entail its conclusion.

1. **{Premises}** validly entail **Conclusion**.

Our justification of indirect deduction was that a valid argument is one whose Counterexample Set entails some sentence and also that sentence’s negation. From (1), and the above two principles, prove a particular version of that claim: that a valid argument is one whose counterexample set entails **(a) the conclusion of the argument, and also (b) the negation of that conclusion**.

a) **{Premises, Negation of Conclusion}** validly entail **Conclusion**

b) **{Premises, Negation of Conclusion}** validly entail **Negation of Conclusion**

¹ In 2.19.